LEIBNIZ’ DYNAMICAL METAPHYSICSAND THE ORIGINS OF THE VIS VIVA CONTROVERSY*

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Although recent work has begun to clarify the later history and development of the *vis viva* controversy, the origins of the conflict arc still obscure.[1] This is understandable, since it was Leibniz who fired the initial barrage of the battle; and, as always, it is extremely difficult to disentangle the various elements of the great physicist-philosopher’s thought. His conception includes facets of physics, mathematics and, of course, metaphysics. However, one feature is essential to any possible understanding of the genesis of the debate over *vis viva:* we must note, as did Leibniz, that the *vis viva* notion was to emerge as the first element of a new science, which differed significantly from that of Descartes and Newton, and which Leibniz called the science of dynamics. In what follows, I attempt to clarify the various strands which were woven into the *vis viva* concept, noting especially the conceptual framework which emerged and later evolved into the science of dynamics as we know it today. I shall argue, in general, that Leibniz’ science of dynamics developed as a consistent physical interpretation of certain of his metaphysical and mathematical beliefs. Thus the following analysis indicates that at least once in the history of science, an important development in physical conceptualization was intimately dependent upon developments in metaphysical conceptualization.

1. *DYNAMICS vs. MECHANICS*

The first thing which we must realize about the origins of the *vis viva* controversy is that it originated because Leibniz was attempting to lay down the fundamental axioms and definitions of a new science— a science of forces in motion “for whose explanation I have set up a distinct science of dynamics”.[2] Leibniz was well aware that he was coining a name for a new and distinct branch of physics. His new science, in his view, concerned bodies active in motion, doing work and producing power. Leibniz’ dynamics, to his own mind, was distinct from the more common and familiar physics of statics—a science well-known since Archimedes’ classic work on the subject. Moreover, Leibniz' science was distinct from the simple physics of motion, as exemplified by Newton's *Principia.* It is true that Leibniz' dynamics needs at bottom something like the basic Newtonian laws of motion. But, in addition, he needed several axioms regarding the ways in which moving bodies behaved when engaged in doing work. That is, he needed primary laws describing the way in which energy was stored, transformed, and expended by interacting bodies. Obviously, this new science was a complicated one. Moreover, in no sense can Leibniz be thought to have been duplicating Newton's work. I have sometimes heard the criticism levelled at him that the *Specimen* was a plagiaristic effort to cash in on Newton's newly-published *Principia.* Such a criticism is misguided. It is true that Leibniz had recently read Newton, and that laws of motion are fundamental to any science of dynamics. But laws of motion are not *sufficient* to generate a science of dynamics. Rather the case may be made for the independence of dynamics and kinetics. What must be added to the basic kinetics is a conceptual scheme in which individual bodies have energetic states and consequently can produce active work.

The differences between the Leibnizian science of dynamics and the Cartesian-Newtonian mechanics are so significant, and essential to preserve, that I have made a terminological adjustment in an attempt to keep things clear and distinct. In the following account, wherever Leibniz uses the term *'vis'* I have not translated it by its usual form 'force'. My reason for this is simple. 'Force', as a term denoting a particular physical concept, is legal tender in modern physical talk. But the modern connotation simply won't do when the discussion centres upon Leibniz. Our contemporary usage of 'force' is decidedly and pervasively Newtonian, with slight overtones of a quantum-relativistic sort. Leibniz' term *'vis',* however, is in no way synonymous with our contemporary term 'force'. *'Vis'* is not even very *similar* to the original Newtonian conception, as Leibniz himself was well aware.[8] The differences in the usage of the term *'vis'* which we see between Descartes (who, in fact, used the term rarely), Leibniz, and Newton are deep- seated, and due to the differences in their physical and conceptual system. Let us note briefly some of these differences since they are of importance for the history of physics.

Descartes and Newton are quite close in what they are trying to explain. Their aim was to set up a rational mechanics, a geometry of the physical world. It is perhaps clearer in Descartes what this amounts to in actual practice. In the conceptual underpinnings of Descartes' mechanics, extension plays the chief part. If matter is purely extension, then the laws of moving[4] matter will be purely geometrical. Motion, moreover, will be a simple spatial displacement in which direction, acceleration, inertia, and so on will play no part. Motion thus becomes a purely scalar quantity, a sort of shadowy ‘quality’ which is superadded to geometrical bits of extension. This is the ‘rational’ element in mechanics—the pure, clean geometrization—which has no concern for the internal states, resistances to motion, energy levels, *et al„* of the piece of matter under consideration. Descartes’ mechanics, a pure kinematics, is phenomenalistic, external, in a manner in which Leibniz’ dynamics could never be.

Newton in this case is merely a more sophisticated Descartes. Again, his mechanics is a pure kinematics, an external, isolated observation of things in motion. True, Newton does grasp the inertial properties of matter. But his concern, like that of Descartes, is not with the internal, ever-changing states of the moving thing itself. Newton’s measures of force—F=ma—takes account of acceleration as a primitive element. But what is acceleration but a reified quality produced in a moving body by an external cause? The Newtonian force- of-a-thing is not a pure dynamical quality of the thing in question; rather, it is a kinematic notion which allows prediction to be accurately made. Moreover, as Mach has shown,[6] Newtonian force may be eliminated from mechanics; thus is eliminated the only faint vestige of true dynamics from the Newtonian scheme. Newtonian force is a construct, a conceptual means to open the only door preventing mechanics from becoming statics. Rational mechanics, for both Newton and Descartes, *is* nothing other than an extension of statics.

What assures Leibniz’ major role in the evolution of physics, as Dugas[6] justly notes, is not merely his bridging of the gap between statics and dynamics. More significant is the *method* of the bridging: his introduction of purely dynamical concepts into physics. Motion, for Leibniz, even though only a *phenomenon bene fundatum,* signifies a real thing in a manner entirely alien to either Descartes or Newton. Motion signifies an internal state of an existing entity. If motion ensues, changes, or has vectorial quantities, it is because some corresponding real state of an entity is itself in the process of change. Moreover, in opposition to the Newtonian view, motion (or any of its processes) is internally caused, rather than being purely externally localized. Only in the Leibnizian system can the motion itself—or the real state that it signifies—be the cause of some subsequent state. This is because every prior monadic state necessarily leads to some later state. Only this essentially dynamic attitude allows the development of concepts such as forces or fields.

Leibnizian force, which I shall call *vis,* comes so close to being a direct precursor of certain entities in modern physical theory, that in some ways it is unfortunate that Newton's physics took such dogmatic doctrinal hold. Leibniz' doctrine of the conservation of *vis viva* is quite correct, and led directly into the principle of the conservation of energy, and the notion of entities being in "particular energy states". His intuition of physics, with its inclination to a dynamical viewpoint, worked to good effect here, and in the fifth letter to Clarke, he hits on the one piece—transformation of energy types—which was still missing from a clear statement of the principle of the conservation of energy. This dynamical, internally energetic aspect of physical bodies is the point is Leibnizian *vis.*

It is for just such an aspect that Leibniz' metaphysics provides the foundations. His whole notion of *vis* derives directly from his metaphysical view of substances:

> The importance of these matters will be particularly apparent from the concept of substance which I offer. This is so fruitful that there follows from it primary truths . . . —truths heretofore known in part though hardly demonstrated, and unknown in part, but of the greatest utility for the future in the other sciences.[7]

This concept of substance is in itself his first criticism of the fundamentals of the Cartesian mechanics. We know this concept from other works: corporeal substance is constituted by active and passive force. Against the Cartesians, this doctrine denies that the essence of material bodies is extension:

> We have suggested elsewhere that there is something besides extension in corporeal things: indeed, that there is something prior to extension, namely, a natural *vis* everywhere implanted by the Author of nature . . .[8]

The Cartesians had held that extension—the purely geometrical— constituted the essence of corporeal body. But, as Leibniz discovered, this conception, if taken seriously in physical experimentation, entails severe problems: and he went to great length to point out these empirical problems to the Cartesians. But his reasons for adopting a scientific metaphysics of force were not entirely empirical. He also had more philosophical reasons:

> Indeed, it [vis] must constitute the inmost nature of the body, since it is the character of substance to act, and extension means only the

> continuation or diffusion of an already presupposed acting and resisting substance.[9]

The kernel of this comment is the claim that "it is the character of substance to act". From this basic belief it is surprisingly easy to reconstruct Leibniz' reasoning, given what we know from other works.

Leibniz firmly held to the belief that every metaphysical substance always acts. Moreover, as we have seen previously, corporeal substances —the entities of physics—are *phenomena bene fundata.* Consequently, it follows that corporeal substance itself always acts and, hence, that *vis* "must constitute the inmost nature of body". This progressive a priori argument, however, is not the only one which is compressed into the statement above. The second part of the statement displays the regressive argument: "extension means only the continuation or diffusion of an *already presupposed* acting and *resisting substance".* Leibniz' argument here is typical and fascinating: to his mind, extension was not a simple idea, but an analysable one, which terminated in *vis* as the surd.[10] Leibniz was quite sure that the well-founded passive powers of bodies—resistance to motion, impenetrability, etc.—were more essential than simple extension. His regressive argument from bodies to *vires* shows the simple character of the step. It is as if he were asking "On what do we ground our observation of extension?" and answering himself by saying "An already presupposed acting and resisting substance." That is, since extension is not a simple idea, it may be analyzed into simpler notions. It then becomes a property which is dependent upon some *other* property of substance. And this line of dependence ends nowhere else but in the *vis* states of substances. It cannot be over-emphasized how important is this departure from Newtonian and Cartesian views. Rather than constituting the essence of a material substance, extension becomes only a mark, a sign as it were, of the fundamental activity of the body. Extension is the limiting field of an energetic, acting thing—its activity itself being diffused from a centre and, it seems clear, *causing* observed extension. With this about turn in the conception of extension and material bodies, Leibniz, however prematurely, ushered in the basic notions of field physics. Moreover, he prepared the way both for his later critique of Newtonian absolutism, and the subsequent dynamics of Wolff, Kant, and, of course, Boscovich. But what is more important for our present purposes is the connection between the new anti-Cartesian conception of substance and of *vis,* and the empirical problem I mentioned above.

2. *METAPHYSICS AND CONSERVATION*

In Cartesian mechanics, an empirical difficulty is encountered immediately extension is taken to be the essence of substance. The basic properties of bodies in interaction—inertial motion, for example—seem not to follow in any straightforward way from the purely geometrical rules, which they should do if the essential property of material bodies is extension. This means that the Cartesians, given only extension, did not have an adequate metaphysical niche in which to locate the grounds of, say, the inertial properties of matter. Their empirical results clearly illustrate indecision about what exactly to do with the inertial properties of matter. Leibniz, in a famous essay, noted this Cartesian difficulty and made much ado about it. The essay, *A Brief Demonstration of a Notable Error of Descartes and Others Concerning a Natural Law,* is itself notable, in fact notorious, in that it initiated the *'vis viva'* controversy.[11] Mach claims that this controversy was based upon "various misunderstandings" and "lasted fifty-seven years, till the appearance of d'Alembert's *Traite de dynamique* in 1743."[12] The 'misunderstandings' to which Mach here refers, have, in my opinion, never been clearly laid out. The *vis viva* dispute is crucial for our understanding of both Leibniz' critique of the Cartesians, and the positive content of his physical doctrines. Leibniz' notion of *vis viva* follows directly from his conception of active and passive force in the monad. Consequently, *vis viva* is consistent and well-founded in his metaphysical system. However, his method of arriving at and proving the *vis viva* doctrine has an empirical basis. It is a moot question, whether he developed the metaphysical concepts first, and then happily managed an empirical method of proof, or whether the empirical element came first. The doctrine can be sorted out without settling this.

To start with, both Leibniz and Descartes were certain that some physical quantity was conserved in the world of matter-in-motion. Their reasons for this belief are many and varied, but may be divided into two categories: metaphysical and methodological.

The metaphysical reasons the two men offer are basically similar. Both grounded in the nature of God their belief that some quantity intimately connected with matter-in-motion was conserved in the physical world, Descartes construed conservation as a result of the constancy of God: the amount of motion in the universe remained constant because God had created an initial amount, and since His own nature remained constant the basic mode of physical nature—motion—should also remain constant.[13] But Descartes was severely constrained in his choice as to the physical interpretation of this conserved quantity. In the first place, Descartes did not have a sufficiently rich metaphysics of matter to allow him to give a consistent account of the non- extensional properties of matter. Secondly, he had no adequate notion of motion himself. Specifically, Descartes did not consider direction as an element of *motus:* 'motion' for him was simple spatial displacement, non-dynamical translation of extended bits.[14] Hence simple speed was his only quantification of motion. This results in a completely non-vectorial (i.e., simple scalar) analysis; analysis of this limited sort complicates problems involving collision, circular motion, and changes of direction nearly beyond the possibility of solution.[15] In these respects, Descartes' choice of an interpretation of *motus* which would serve as a conserved quantity was limited from the very first. His choice was simply 'quantity of motion'.

Descartes' 'quantity of motion' was a simple quantity, the product of 'bulk' and 'speed'; or, in modern terms, a quantity akin to momentum. Thus, when Descartes claimed that the quantity of motion in the universe remained constant, he was claiming that the amount of momentum in the universe remained constant. The unfortunate fact— unfortunate, that is, for the subsequent *vis viva* dispute—is that Descartes was correct, but only in a qualified range of cases; his belief was quite wrong in another range of cases. It was, of course, just these cases that Leibniz picked to criticize Descartes' mechanical law of conservation.

Leibniz' metaphysical reasons for believing in a conserved quantity were similar to those of Descartes: belief that the universe as a whole retained its summation of properties. The grounds of this were, like Descartes', metaphysical: Leibniz believed in the constancy of God's properties and the high quality of God's creative abilities. He reasoned that if the basic properties of the universe were not conserved, the universe would run down. If this happened God would have to tinker with it in order to keep it functioning properly. But isn't it obvious, reasoned Leibniz, that God was a better workman than that—wasn't it within His power to create a maintenance-free universe? Obviously, God has such power. It follows then that some basic property is conserved.

Which property was conserved Leibniz found easy to decide, since his metaphysics functions solely in terms of individual monadic actions and passions. Correspondingly, on the theoretical, explanatory level of the primary corporeal substances of physics, the world functions solely in terms of active and passive *vis.* Finally, the correspondence of the secondary or phenomenal level is then in the observable *vires* of resistance and activity. According to this three-level philosophical system, Leibniz the physicist could choose only *'vis'* as his conserved quantity.

This simple move from Leibniz' metaphysical level, through the explanatory level, to the phenomenal level strikingly reveals by comparison the deficiency of Descartes' metaphysics. Descartes' metaphysics was too limited to allow this sort of move. For him the fundamental property of corporeal substance was extension. But extension is not an active property; it is simple and purely passive. In the Cartesian mechanics the initial conserved quantity is, and can be only, some fundamental mode of extension. Motion and mass, as modes, are doubtless intimately related to extension, but how? Motion is clearly a mode of material being, and it can be quantified. But what is the relation of motion and mass in momentum? In the Cartesian physics these questions are confounded, puzzling, and confused. In a physics which, like the Cartesian one, lacks an adequate notion of the relation of mass to extension, it is difficult enough to reach a conception of simple momentum, let alone a clear conception of *vis viva.* Leibniz' conception of active material constituted by states of *vis* makes it much easier for him to ground the requisite notion of energetic states of matter in motion. Leibniz' more highly developed metaphysics certainly was of assistance in his decision regarding the choice of a conserved quantity.

But these metaphysical reasons behind the *vis viva* dispute are not the only reasons for the disagreement between Descartes and Leibniz. There are other, methodological, reasons. The basic methodological reason behind the belief of Leibniz in the conservation of some elemental physical property is simple: if there is no conserved quantity, then it is not easy to describe the physical universe mathematically and in equations.

Leibniz was well aware of this, as is illustrated by his use of mathematical equations to show the inappropriateness of Descartes' choice of a conserved quantity. Since Descartes' quantity did not result in balanced equations, this seemed reason enough for Leibniz to rule out Descartes' choice of a conserved quantity.

At this point we should be clear about the broad outlines of the *vis viva* dispute. For various reasons, both metaphysical and methodological, Descartes and Leibniz believed it necessary to identify a physical quantity which would be conserved in the interactions of the phenomenal world. For much the same sorts of reason, then, the two men made their choices: 'quantity of motion' on the one hand, and *'vis viva'* on the other. But Leibniz showed that Descartes' interpretation could not be generalized to cover every sort of physical situation. By his use of a particular sort of physical problem he refuted Descartes' contention, and substituted a notion of his own choosing. What we must remember, though, is that this argument occurs at the formative stage of a new science, during which each worker attempts to formulate correctly the basic concepts. Hence we must view the argument between them as a necessary prolegomenon to the eventual establishment of the new science. These are the broad outlines of the beginning of the *vis viva* controversy. We are now ready to look at the particulars of the Leibnizian case.

3. *THE EMPIRICAL ARGUMENT*

Leibniz' argument initially looks like a *gedankenexperiment.* However, in several of its versions it might easily be transferred to the laboratory bench. While we are not sure that Leibniz actually performed the experiments, this is unimportant since the descriptions of the experiment illustrate all requisite practicality. The basic schema of the argument occurs many times in Leibniz' writings and correspondence. My quotations are from: *A Brief Demonstration;™ Critical Thoughts on the General Part of the Principle of Descartes;*[11] and *Specimen Dynamicum.™ Brief Demonstration* is both the earliest and the most polemical; in it Leibniz very clearly sets out his assumptions and shows precisely how the argument refutes the Cartesian case for the conservation of momentum. He starts the refutation by stating the case for a principle of conservation:

> Now since it is reasonable that the same amount of motive force should be conserved in nature and not be diminished—since we never see *vis* lost by one body without being transferred to another—or augmented, a perpetual motion machine can never be successful because no machine, not even the world as a whole, can increase its *vis* without a new impulse from without.[19] This led Descartes, who held motive force and quantity of motion to be equivalent,[20] to assert that God conserves the same quantity of motion in the world.[21]

Following this justification of the search for a conserved quantity, Leibniz begins to lay out his argument against the Cartesian choice. He begins with a simple principle:

> I begin by assuming . . . that a body falling from a certain altitude acquires the same *vis* which is necessary to lift it back to its original altitude if its direction were to carry it back and if nothing external interfered with it.[22]

This initial principle is crucial. Two points concern us: one, the example focuses on the weakest link in the Cartesian case. As I mentioned earlier, Descartes' 'quantity of motion' is not conserved in cases which involve changes in direction of the body in question. Thus Leibniz, given his first claim, can give a satisfactory answer in three types of physical cases with which Descartes cannot deal at all: circular motion, where change of direction is constant; rebound cases, in which an imagined body possesses perfect elasticity; and, finally, exchange of energy, such as that of one body propelling another from a see-saw. He eventually uses cases of all three types.

The second point which concerns us in this first principle clearly indicates the source of Leibniz' guiding inspiration, for he goes on to say: "a pendulum would return to exactly the same height from which it falls."[23] Descartes, in his laws of motion, does not mention the pendulum. An important type of moving body, the pendulum is useful to Leibniz here, since it is a classic instance of circular motion. It is also easy to observe; hence Cartesians cannot deny the truth of his claim. Already at this early stage of the argument, the only way the Cartesians could evade the consequences would be to point out that momentum was not conserved in circular motion—but this would be to admit the deficiencies of their notion. Moreover, this fact was not known by them prior to the settlement of the *vis viva* dispute—the very point at issue here! In any event, even at this preliminary stage, the Cartesian case for conservation of momentum is doomed. Leibniz' second claim merely seals the coffin more tightly:

> "I assume also, in the second place, that the same force is necessary to raise the body A of one pound to the height CD of 4 yards as is necessary to raise the body B of 4 pounds to the height EF of 1 yard.[24]

The letters refer to Figure 1.

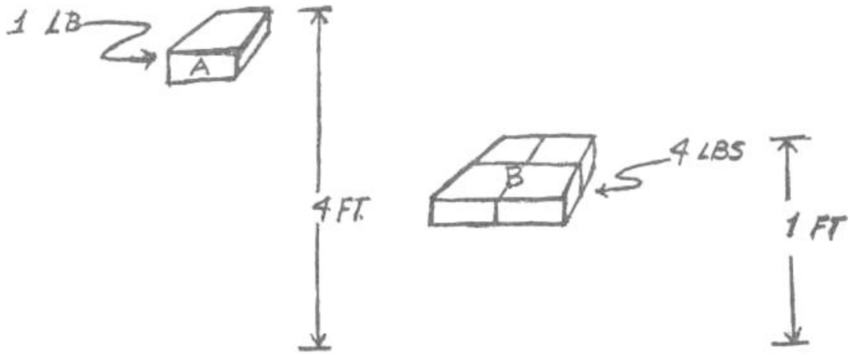

Figure 1

Leibniz' second assumption is that the forces involved in lifting the two bodies through the respective distances in the same time are equal. It is so easy to see this, that he claims that "Cartesians as well as other philosophers and mathematicians of our times admit both of these assumptions."[25] Leibniz now has all that he needs to prove that the Cartesian case for the conservation of momentum is false; to prove, that is, that the proper interpretation of motive force is not quantity of motion, but rather Leibnizian *vis.* At this point in time (1686) he does not tell us what exactly the proper mathematical interpretation is—he may not have known it yet. But he does tell us what it is not: it is *not* Cartesian quantity of motion.

The argument proceeds as follows: we know that the motive forces acquired by bodies A and B while falling their respective distances are equal. On the Cartesian interpretation of motive force, this would mean that the quantities of motion of bodies A and B must be identical, if motive force is to be conserved. Quantity of motion is the simple product of mass and velocity. In its fall, the body A of mass 1 acquires the velocity 2.[26] Hence its momentum is mv=(1 x 2)=2. In its fall, the body B of mass 4 acquires velocity 1. Hence its momentum MV=(4 x 1)=4. But the Cartesian principle of conservation of quantity of motion claims that mv=MV, if the respective motive forces are to be equal. Unfortunately mv=2 and MV=4, and 2 is not equal to 4. Ergo, the Cartesian case for conservation of momentum cannot be made out. At this point, admittedly, Leibniz is unsure of the correct mathematical description of his measure of *vis.* But he clearly does know what sort of thing it will be a measure of: it will be some measure of the *effect* which moving bodies can produce; 'effect' being their ability to do work,[27] e.g., lift themselves after descent. As Leibniz tells us:

> It seems that *vis* is rather to be estimated from the quantity of the *effect* which it can produce; for example, from the height to which it can elevate a heavy body of a given magnitude and kind, but not from the velocity which it can impress upon a body.[28]

This method of arriving at the measure of *vis* through its effect is one of Leibniz' prime contributions to the physics of his time. It is true that interest in this sort of measure was current during the period, but Leibniz' careful use of the principle in this particular example is germinal to the later development of the physics of energy. As a matter of fact, his first assumption, that a falling body preserves the same *vis* through the zero point of directional change, i.e., that in its fall it acquires *vis* sufficient to regain its altitude, shows exactly the method of measuring *vis* by its effect. Moreover, it also eliminates one of the variables from candidacy as an element in the calculation of real *vis:* he states that *vis* is not to be calculated "from the *velocity* which it can impress upon the body" (italics mine). Simple velocity as an element of *vis* computation is thus eliminated, since it is not proportionally related to the total effect produced by a *vis.* As I have mentioned, Leibniz at this stage does not know exactly how to describe mathematically the total effect produced by a *vis.* But he does know two important ingredients in the calculation: the magnitude, or mass, of the body; and some measure which relates to the altitude of ascent—the height raised against gravity. This latter measure might even be some *complex* measure of velocity (as we shall see, it is), but it is not simple velocity. Even given these two elements, the concept needs further development. We do not know precisely how the development went in Leibniz' own mind, but we can certainly make a plausible reconstruction of the evolution from this point, given what information we do have.

The next stage of development in the *vis viva* concept can be seen in Leibniz' 1692 essay, *Critical Thoughts on the General Part of the Principles of Descartes.* In article 36 of this essay, Leibniz first runs through exactly the same example which we saw above in *Brief Demonstration.* Then he moves into a further case, one in which the force of falling body A propels the subsequently ascent of body B. (This is the third sort of case involving change of direction. We can imagine B sitting on the ground end of a see-saw, and A falling from some height on to the upper end of the board, thus propelling B skyward.) Now, supposing that either the whole or some part of the *vis* of A is transferred, is the Cartesian quantity of motion conserved or not? Leibniz answers:

> Even if a part of the *vis* is retained by A and only a part transmitted, the same absurdities would still arise, for if the quantity of motion is to be conserved, the quantity of *vis* can obviously not always be conserved, since the quantity of *vis* is, as we have shown, the product of mass and the altitude to which it can be raised by force of its power, altitudes being proportional to the square of the velocities of ascent.[29]

Compressed into this comment is the entirety of the *vis viva* dispute, and its Leibnizian solution. The first element that we can unpack is one which we have seen previously, but from a different aspect. On the supposition of the conservation of momentum, the quantity of *vis* cannot be conserved. The case goes like this: Body A has a mass of 4 and a velocity of 1. Its quantity of motion is mv=4. Body B has a mass of M = l. What will be the velocity of B if the *vis* of A is transferred to it? If, on the Cartesian supposition, mv=MV then 4 x 1 = 1 x V, or, B receives a velocity of 4. At this point Leibniz notes that perpetual motion results, since the original alititude of A will be surpassed by B, if B's velocity is in fact 4. That is, mass times altitude is the measure of *vis.* And altitudes of ascent, by the above quote, are "proportional to the squares of the velocity of ascent". Thus the altitude reached by B will be proportional to $V^2=(4)^2$, or 16! This 16-fold increase in altitude clearly contradicts the assumption of proportionality of altitudes for similar *vires.* This means that perpetual motion results if one assumes the conservation of momentum in this particular case.

Leibniz' remark regarding the relation of altitude to squared velocity is the final necessary element: the proportionality of altitude to squared velocity will produce the correct mathematical estimation of Leibnizian *vis.* In *Brief Demonstration* he had cast aside *simple* velocity as not being directly related to *vis,* and had settled on altitude as a proper measure. Unfortunately, altitude is not a simple dimensional quantity us arc, for example, mass, time, and distance. Altitude is compounded with gravity, and distance of fall. But Leibniz, in this rather off-hand comment that the altitude of ascent is proportional to the square of the velocity, indicates that by 1692 he was well on the way to a correct mathematical settlement of the measurement of *vis.* What he needed now was to arrange all the elements into the appropriate equation. But then the final arrangement would also need integration into his metaphysics.

He did not get it entirely right until *Specimen Dynamicum* in 1696. But first we must take at least a brief look at the mathematical complexities which Leibniz had to deal with on the way to the final correct definition of *vis viva.* Otherwise we shall not understand what is going on in *Specimen.* The problem is exceedingly involved: In the first place, Leibniz knew that falling bodies could be usefully studied with respect to their motive *vis.* Note that falling bodies *per se* are not of interest to him, only the *vires motrices* involved. The major difficulty was to set up simple measures which would allow one to isolate and abstract the element of motive *vis* from the falling body cases. The simplest measures are obviously distance and mass. Time may be indirectly computed from other easier measurements, altitude being one of these. Using Galileo's law, one may easily compute the time involved in a body's fall.[30] From Galileo's law

(1) $s^2 = 1/2\ gt^2$, where s= distance (altitude), g=gravitational attraction, and t=time of descent, it is a simple substitution to get

(2) $t^2 = 2s/g$

Now one knows the time involved, and the velocity is easily computed from the further law that

(3) $v = gt$, where v=velocity.[31]

But then, we saw in the passage in *Critical Thoughts* that Leibniz mentioned that the altitude of ascent of a propelled body is proportional to the square of its initial velocity. This conclusion appeared on its own, without any justification. How did he reach it? The first thing he needed to do was to eliminate the time measure, since his only fixed, accurately measureable factor was distance. (Another reason for eliminating time was his Cartesian heritage of preference for distance measures in physical problems.) The elimination of temporal variables can be accomplished by first squaring (3) in order to get in equivalent powers as (2). Thus,

(4) $(v)^2 = (gt)^2 = v^2 = g^2\ t^2$.

Then, substituting the right-hand side of (2) for the t^2 of (4), one gets the required solution, that is, the solution purely in terms of gravitational attraction and distance:

(5) $v^2 = g^2\ 2s/g = v^2 = 2gs$.

A final manipulation produces the solution in easily measured, purely spatial terms:

(6) $s = 1v^2/2g$

From this series of equations, solved only for easily and accurately measurable variables, Leibniz (in our reconstruction) has the mathematical description to suit his case: since the altitudes of ascent of falling bodies are proportional to their *vires* and their altitudes are also proportional to the square of the velocities of their ascent, it follows that *vires* are proportional to squares of velocities. But there seems to be a remaining perplexity: how do all these factors fit together to give a coherent interpretation of Leibnizian *vis?*

4. *THE FINAL INTEGRATION*

Specimen Dynamicum has the complete answer. Additionally, in strong contrast to *Critical Thoughts,* it presents the solution *in full integration* with the metaphysical system. This seems to indicate that in the particular case of *vis viva,* Leibniz worked out the physical parameters before developing the metaphysical elements. For example, in *Discourse on Metaphysics* (15-18), 1686, he specifically refers to both the actions and passions of monads (metaphysical levels), *and* the forces of physical bodies (phenomenal level), but he does not relate them in any clear and precise way. What was lacking was the conceptual development *linking* the phenomenal level to the elementary metaphysical level. To establish this connection, Leibniz had first to resolve several puzzling problems: what was the explanation of the ever-present influence of gravity in all the equations? what underlay the apparently purely contingent relation of the square of velocity to the total effect? and why was it necessary to include the magnitude (mass) of a body in the final mathematical estimations of secondary phenomenal *vires?* Leibniz studied all these problems, and more or less satisfactorily resolved them.

In order to understand the manner in which he resolved them, and subsequently linked his metaphysical system to the physical parameters, we should first recall some earlier points. First, every corporeal substance possesses its own *vis.* This *vis* is independent and exists entirely *within* the body. The use of 'within' here has a complex ambiguity which is the key to the Leibnizian dynamical attitude.

Logically, 'within' may mean 'within as in class membership', so that *vis* is one of the members of a set the whole of which we call 'body'. Literally, 'within' may mean 'on the inside of, so that it implies that there is an outside. Metaphysically, 'within' may mean ‘belongs to’, so that something which inheres is an element of that to which it inheres. But the connotation of metaphysical inherence is inappropriate. Although the term ‘property’ is often used to refer to the primitive active *vis,* it must be understood in its Leibnizian sense. That is, Leibnizian substances are *not* substrata *plus* properties—the pincushion and pins analogy—rather he holds that the sum total of properties *constitutes* the substance.

The notion of ‘within’ here is a new conceptual development, which explains to some extent why Leibniz’ dynamics differs from Newton’s kinetics. As far as physics is concerned, this new metaphysical attitude has enormous ramifications, since it results in the fundamental conception of a system of dynamics. From this viewpoint, a corporeal substance does not *have* certain states of *vis,* as though they were something added to an already existing substratum; no, a corporeal substance *is* a succession of certain states of *vis.* This interpretation implies that corporeal substances are active in themselves and by themselves, and hence must be considered to be internally active and efficacious. On this interpretation every corporeal substance is a force and energy centre necessarily involved with every other substance. The consequences of this view for field dynamics are obvious. If matter is, in some cases, energetic and alive, then the purely kinematic notion of dead bits moved from without is a false conception. The notion of ‘within’ which allows this richness of Leibnizian dynamical connotation is the ‘within’ I intend here when I claim that *vis* is within each body.

But *vis* itself is of two types, active and passive. Passive *vis,* considered as phenomenal, may be interpreted as a resistance.[32] Thus ‘mass’ as a passive force is the measure of inertial resistance.[33] Moreover, mass is an additive property: as the number of substances in an aggregate increases, so also does the mass of the aggregate. But as we have seen, Leibniz claims that mass is one of the independent variables in the computation of *vis viva,* and this is an inconsistency: if mass is only a *passive* property, how then does it enter into the computation of the active living force, *vis viva*? Leibniz does not seem ever to remove this stumbling block. It is not even discussed—rather, Leibniz always speaks of derivative *vis* “or the *vis* by which bodies actually act and are acted up by each other” as “only that *vis* which is connected with motion (local motion, that is) and which in turn tends to produce further local motion”.[34] He thus seems to combine both active *and* passive derivative *vis* in the *concept* of *vis viva.* However, in the *measurement* of *vis viva,* he uses only the passive *vis* of resistance as a parameter. This asymmetry is disturbing; it represents a definite deficiency in the metaphysical grounding of his physical system.

But the notion of mass at least is sufficient to allow one to isolate independent corporeal *vis* from the physical cases we examined above. As I said earlier, gravitational attraction appears as a constant in the mathematical descriptions of falling bodies. But for Leibniz gravity is *not* an inherent property, or *vis,* of bodies, since he always claims that it is an external, mechanical influence which operates solely by physical contact.[35] Further, since Leibniz is trying to describe that *vis* which is self-contained in bodies, gravity, as an extraneous influence, must be eliminated from the mathematics. The concept of 'mass' allows this to be carried out. Mass is a *vis* of resistance. But resistance to what? On Leibniz' strange theory of gravity, it is a resistance to gravitational 'attraction'. This implies that the greater a body's resistance to gravity, the greater its mass. Because of this simple direct proportionality, we can eliminate the gravitational constant from the equations. What allows us to do this is the fact that in all physical cases, gravity was operating equally upon the *varying* masses; consequently, the operative independent variable which was *directly* related to the different forces was the mass of the bodies themselves. In other words, in the physical cases the determining factors were the varying amounts of passive *vis* which were actuated by the equal gravitational 'attraction'.

One point must be noted, however. Given his sophisticated dynamical attitude, it is strange that Leibniz did not notice the peculiar coincidence of gravitational and inertial mass. This might be explained by realizing that on the whole his theory of gravitation remained crude and naive in contrast to the subtlety of his dynamics. Gravitation remained a separate and distinct influence upon bodies, and all explanations of gravitational 'attraction' were couched in purely mechanical terms. His theory countenances the notion that heaviness of bodies is due to inertial resistance, but this resistance is not a field-type interaction. Rather, it is purely mechanical. However, even *given* the crudeness of the theory, it is surprising that he never noted the equivalent resistances to acceleration in both the gravitational and purely spatial fields. An explanation of this deficiency in his system is found in the inadequacy of his theory of gravitation. On the other hand, *had* he noticed the puzzling equivalence of the two mass quantities, Einstein would have been two centuries too late.

There remained only one factor to be resolved by integration into the scientific metaphysics: why was squared velocity a vital element in the measuring of *vis viva*? We have earlier seen the physical reasoning by which this variable was initially discovered; namely, the proportionality of the altitude of ascent to velocity squared. But what is metaphysically parallel to this reasoning? The answer takes us into the far reaches of Leibniz' mathematical probings into the intricacies of the differential and integral calculus which had a substantive influence on his metaphysics. It seems clear to me that such an influence is also present here in this stage of his dynamics.

From the concept of infinitesimal analysis in the differential calculus, Leibniz had evolved a notion of infinitesimal metaphysical points comprised by infinitesimal amounts of *vis.* In dynamics, infinitesimal points are interpreted as corporeal substances, while infinitesimal *vires* are interpreted as instantaneous velocities or '*conatus*'[36] Thus each and every corporeal body has at any instant of time its own instantaneous (vectorial) velocity.[87] Here again it is essential to remark on the deep-seated conceptual difference existing between the Cartesian- Newtonian and Leibnizian interpretations. For the Cartesian-Newtonians, both velocity and instantaneous velocity are averaged quantities. For the former, the average is of the space traversed over time; for the latter it is the averaged velocity. On these views, what is measured is the *space* traversed, and thus velocity is not intrinsically related to the bodies themselves.

The Leibnizian view differs in two ways. First, the instantaneous velocity is not interpreted as being merely the average velocity, but is seen rather as a measure of the activity, the *conatus,* of the body itself. Thus instantaneous velocity is the measurement of a body's tendency to do something, of its activity "pregnant with past and future". The second consideration is this: In the Leibnizian view space is constituted by existing bodies in the sense that it is merely the simultaneous relation of all juxtaposed existences. This means that space is logically and metaphysically dependent upon bodies; consequently, instantaneous velocity, while it appears to be a spatial measurement, is actually a measurement of some activity of these bodies and that they produce spatial relations. This collapsing of spatial relations into intrinsic states of bodies is possible only on purely dynamic grounds. It would be hard to find a more clear-cut indication of Leibnizian-Newtonian differences than those seen here in the interpretation of instantaneous velocity.

But beside velocity, another factor is needed to describe the instantaneous *vis* of a corporeal substance. A secondary phenomenal body, for Leibniz, is an aggregate. But the relevant measure of the magnitude of a aggregate body is nothing other than the measure of the mass of that body. This means that mass must enter into the computation. Consequently, for example, the instantaneous measurement of the *vis* of an aggregate body is its mass times its instantaneous velocity. But at this point Leibniz moves from differentials, and puts to use the complementary part of his calculus—integral calculus. The reason for this move is fairly simple. *Vires* always act through time.[38] The measures of *vis* which we have discussed until now have all been instantaneous measures, which are essentially atemporal. So, in order to measure the *vis* of a moving body, we have to integrate the infinite number of infinitestimal impetuses which are occurring in a given interval to the aggregate body. Of course, as I have just indicated, to do this we have to integrate the instantaneous velocities from time 0, when the body starts to fall, to time t, when it completes its fall. In addition, this last quantity will have to be multiplied by the number of members of the aggregate, which is given by its magnitude or mass. The final quantity is the Leibnizian *vis viva.* This linking of infinitesimal mathematics and metaphysics with the dynamics of secondary phenomenal bodies solved the puzzle of why the square of the velocity was a suitable empirical measurement of the force of a work-producing body, for the reduction of the differential quantity into second-power algebra is none other than the square of the velocity. The algebraic notation 'v^2' may then be correctly seen as a simpler representation of the more complex, *actual,* metaphysical situation involving infinitesimal forces, these latter being *adequately* represented only by expressions given in terms of differential and integral infinitesimals.

5. *RESULTS OF THE DYNAMICAL ATTITUDE*

At this point Leibniz has all the pieces put together. All the various parts are synthesized into a coherent whole. The initial attack on the Cartesian idea of [passive matter+momentum conservation] has lead to a complete systematic view constituting what I have called the dynamic attitude: infinitesimal, instantaneous *vires,* represented by differential equations, strictly correspond to the momentary internal states of active metaphysical beings. The physical, or *phenomenal,* expressions of the infinitesimal *vires* may be represented by their integrative summation according to the calculus. Nothing could be farther from the static Cartesian-Newtonian view than this living, active conception of material substance.

Moreover, the final formula which came out of this development, that is, the formula stating the principle of the conservation of *vis viva,* had enormous cash value. The principle of the conservation of energy is the direct lineal descendant of Leibniz' principle.[39]

One further dividend of Leibniz' work on *vis viva* must be noted. Leibniz was the first to discover the narrow gap between statics and dynamics, and the first to erect a conceptual bridge between the two domains. The erection of the bridge also tied in completely with the conceptual development of the idea of energy and energy transformation.

The conceptual bridge itself derives directly from the formal aspect of the calculus. The infinitesimal active *vires,* which are represented by differential equations, may be considered, from one point of view, to be instantaneous static forces of the type found in the physics of statics. Note here the subtle change in the interpretation of *static* forces: even they have become active states, which merely are not yet quite sufficient to produce work. On this interpretation statics thus becomes a kind of slow-speed dynamics, as opposed to the Newtonian interpretation in which dynamics is a high-speed statics. Viewed in this way the *vires* of statics may be called, according to Leibniz, "dead *vires*"—the *vis* of statics being called 'dead' "because motion does not yet exist in it but only a solicitation to motion such as that of ... a stone in a sling even while it is still held by the string."[40] It is only when dead *vis* is in actual motion that it becomes *vis viva.* *[1] The transition of dead *vis* into living *vis* is represented by the integration of all the differentials representing the dead *vires.* As Leibniz himself says "the law of statics thus applies to differentials, that of dynamics to integrals".[42] Thus, given the mathematical formalism of the calculus, the movement from statics into dynamics is smooth and complete. Moreover, the concepts involved —dead and living *vis*—are precisely and exactly equivalent to our modern concepts of potential and kinetic energy. Following the realization by Leibniz of the smooth transition and transformation of dead into living *vis* as mediated by the calculus, it was only a short conceptual step to the added realization that energy, or *vis,* may have indefinitely many modes of transition, that is, that *vis* may be transferred and even transformed into many kinds of equivalent states. That he clearly reached this final realization is readily evident in the fifth letter to Clarke, in which he describes the *vis* of a single aggregate body passing to its component parts upon violent contact. (Sec. 99, 5th to Clarke.)

One conclusion does seem to present itself here. Leibniz' dynamical attitude is not a simple thing. It is a complex binding together of conceptual elements from three different fields, i.e., metaphysics, mathematics, and physics. In the preceding part I have in several places indicated what I thought to be the direction of influence, namely: the physical experiments on *vis* precede the linking of physics and metaphysics which is carried out in *Specimen Dynamicum;* although, of course, the *metaphysical* doctrine of forces as instantaneous *vires* preceded this linking. But it would be a mistake to attach major importance to any of these tentatively indicated directions of influence. I say this because it seems to me impossible to sort out accurately what was going on in Leibniz' mind as far as conceptual fertilization is concerned. Several things are obviously clear: for example, the calculus had to be developed prior to its becoming a suitable representation of both metaphysical being such as monads, and physical bodies constituted by active and passive *vires.* But one is simply unable to uncover, from either the essays or the correspondence, whether the calculus was derived *in order to* represent the metaphysics and physics, or whether the latter were developed in order that the calculus might represent them. The three fields were all under investigation by Leibniz at this same time; hence they were all on his mind during this period. What is important to understand, perhaps, is not the precise manner and order in which they influenced one another, but merely *that* they did influence one another in significant ways. The totally systematic, highly innovative dynamical system which Leibniz developed out of the integration of these three diverse fields cannot be understood until one understands that each did influence the whole, and that none of the distinct developments in each field would have been possible without the corresponding developments in the others. This interpenetration of physics, mathematics, and metaphysics certainly was successful, a fact which should puzzle certain contemporary philosophers of science, especially those positivists who believe that science and metaphysics are mutually exclusive.

REFERENCES (Footnotes to Chapter II)

1. This interpretation of the controversy varies from the standard ones. The widespread view is that the dispute was merely "une dispute des mots". The phrase is d'Alemberts', Jean Le Rond d'Alembert, *Traite de dynamique* (Paris: David, 1743), p. xxi, who, although not the first to reach this view regarding the *vis viva* controversy, certainly gets the usual credit for 'solving' the dispute by means of this criticism. Mach, *Science of Mechanics* (Open Court, 1960), p. 365, credits d'Alembert and goes on to claim that the dispute revolved around "various misunderstandings". This view is probably the received view; Mach is not the only one to hold it. Others, for example, include J. Merz, *A History of European Thought in the Nineteenth Century,* Vol. II (New York: Blackwood, 1912), p. lOOn; William Whewell, *History of the Inductive Sciences,* Vol. II (London, 1857), p. 90; J. Desaguliers, *A Course of Experimental Philosophy* (London: W. Innys, 1735-44), Vol. II, pp. v-vi; and Thomas Reid, "An Essay on Quantity", *Philosophical Transactions,* 45 (1748), 505-520.

Gueroult, on the other hand, does not think that the dispute is merely a misunderstanding, although it is "entirely in vain" from the point of view of scientific progress. He locates the point at issue in the fact that the Cartesians and Leibnizians were attempting to compare forces using different types of dimensional analyses, e.g., equal times versus equal spaces. He does feel, however, that the dispute is a valuable introduction to the Leibnizian a priori method; M. Gu6roult, *Leibniz.: Dynamique et Metaphysique* (Paris: 1967), p. 111-118. Other authors also feel that the controversy was more than a case of misunderstanding, but locate its importance in different areas; *viz.,* clarification of the nature of matter, Thomas L. Hankins, "18th Century Attempts to Resolve the *Vis Viva* Controversy", *Isis,* 56 (1965), pp. 281-297; and the relevance of causal hypotheses in physics, Colin Maclaurin, *Account or Sir Isaac Newton's Philosophical Discoveries* (London: A. Miller and J. Nourse, 2nd ed., 1875), p. 144, and so on. Very thorough discussions of the history of the controversy as it developed *after* Leibniz' death may be found in Hankins, *op. cit.,* and L. L. Laudan, "The Vis Viva Controversy, a Post-Mortem", *Isis,* 59 (1968), pp. 131-143. The clearest modern discussions of the origin of the controversy is in Ren6 Dugas, *Mechanics in the Seventeenth Century* (Neuchatel: Editions du Griffon, 1958), pp. 466-483. Dugas himself does not adopt a clear interpretational position on the dispute, although it appears that he feels Leibniz somewhat at fault for starting the argument on the basis of certain misunderstandings and mistakes (cf., for example, pp. 468, 472, 473n). Cf., also, note 22 below.

2. G (=Leibniz, G. W., *Philosophische Schriften,* C. I. Gerhardt (ed.), 7 Vols. Berlin: 1875-90)) IV, 468-9, Loemker (=L. E. Loemker (ed.), *Gottfried Wilhelm Leibniz: Philosophical Papers and Letters,* 2nd Ed. (Dordrecht: Reidel, 1969)), p. 94.

3. *Cf.* Mach's comments, *Science of Mechanics,* p. 365; or Specimen Dynami- cum, GM (=Leibniz, G. W., *Mathematische Schriften,* C. I. Gerhardt (ed.), 7 Vols. (Berlin & Halle, 1849-55)) VI 240-2.

4. *Cf.,* viz. *Principia Philosophiae,* II, xxv-xxix.

5. E. Mach, *The Science of Mechanics,* p. 96.

6. Dugas, *Mechanics in the Seventeenth Century,* p. 511.

7. *On the Correction of Metaphysics,* G IV, 468-9; Loemker, p. 433.

8. *Specimen Dynamicum,* GM VI, 234; Loemker, p. 435.

9. *Ibid.*

10. To De Voider, March 24, 1969, G II, 169; Loemker, p. 516.

11. GM VI, 117-9; Loemker, pp. 296-301.

12. March, *op. cit.,* p. 365. Two problems are involved here: First, there is considerable doubt as to whether d'Alembert in any sense *solved* the controversy when he first suggested that the dispute be shelved on the grounds that it was merely "une dispute des mots". Laudan, *op. cit.,* pp. 132-5, for example, shows that the dispute in fact did not end with the publication of the *Traite.* Hankins, *op. cit.,* p. 297, on the other hand, shows that d'Alembert was not the first to suggest that the dispute would be better ignored.

Secondly, one wonders about the comment of Mach's. Why would he feel d'Alembert's suggestion that the dispute be shelved, and the law of conservation accepted simply as an empirical datum, to be the appropriate one? One need only note the positivistic aspects of d'Alembert's suggestion to be aware of Mach's reasons. D'Alembert's suggestion was based on two grounds. In the first place, he did not want to introduce theoretical causal hypotheses into physics. Leibniz' dynamical *vires* were clearly of this sort. Secondly, certain other problems, such as the need for the introduction of infinite forces to explain the changes of direction rebound-collision, made him quite uncomfortable. Mach, from his positivistic position, is obviously in agreement at least with the first of d'Alembert's grounds.

13. Descartes, *Principia,* Part II, Sec. 48 ff.

14. Descartes did not have an entirely adequate concept of 'mass'; his concept is closest to simple 'bulk', or 'volume of matter'.

15. Cf. Descartes, *Ibid.,* in which Descartes distinguishes between the quantity of movement and the quantity of its *determination* or direction. He uses this distinction to explain the possibility of free will. But since the two quantities are distinguished and separated, conservation may hold for only one without necessarily holding for the other as well. Discussion of this distinction may be found in Ohana, "Note sur la theorie cartesienne de la direction du mouvement", *Les etudes philosophiques* (Paris: 1961), pp. 313-315; and A. Metz, "Descartes et Leibniz, note sur leurs conceptions de la force et du mouvement, a la lumiere de la science actuelle", *Archives de Philosophic,* 31 (1968), pp. 473-476.

16. 1686, GM VI, 117-9; Loemker, pp. 296-301.

17. 1692, G IV, 354-92; Loemker, pp. 384-410.

18. 1695, GM VI, 234-54; Loemker, pp. 435-52.

19. Leibniz' use of the impossibility of perpetual motion is highly innovative. Leonardo and Galileo had both argued that simple machines could not be used to gain energy in a system; hence perpetual motion could not result from any simple machine. For a discussion of the history of the perpetual motion argument, *cf.* Meyerson, *Identity and Reality* (New York 1962), pp. 202-4. Leibniz' addition to the argument is its increased scope. In the above passage he specifically refers to the world-as-a-whole as a machine conforming to the limitations on energy and increase. This was the first time the non-existence of perpetual motion was claimed on such a universal scope. This principle is important to him in several places during the various refutations of Descartes, since he shows that under certain circumstances Descartes' principle of the conservation of momentum, if strictly applied, leads to a theoretical increase of energy in the physical system under discussion. *Cf., viz.,* to Arnauld, Nov. 28, 1686, G II, 77-81, 2nd discussion below, p. 33.

20. This statement is not strictly true. As Huygens points out, Descartes himself had never made any claim as to the equivalence of motive force and quantity of motion, *Oeuvres completes de Huygens,* XIX, MS F (1686), pp. 162-165. The discussion of this error of Leibniz is to be found in Dugas, *op. cit.,* p. 468. The term 'motive force' does not occur at all frequently in Descartes, although, since Leibniz addressed *Brief Demonstration* to "Descartes and others", it may very well be that certain of the 'others' made the equation between motive force and quantity of motion. The Abb6 De Catalan, in his reply to *Brief Demonstration* (G III, 40), seems to accept Leibniz' equation, since he does not argue against it. In any case, it is not important to my interpretation, since what is at stake is the choice of a conserved quality, and not the name it is to be called. Descartes realized that something was conserved, whether it be called 'motive force', or 'quantity of motion'.

21. *Specimen Dynamicum,* GM VI, 117; Loemker, p. 296.

22. *Ibid.*

23. *Ibid.*

24. *Ibid.;* The following example, one of Leibniz' most oft-used, is cited in both Gueroult, *Dynamique,* pp. 111-118 and H. W. B. Joseph's *Lectures on the Philosophy of Leibniz* (Oxford: 1949), pp. 38-44. Joseph's interpretation is similar to mine in several respects, although his final claim is that "what Leibniz is really most interested in ... is the doctrine of substance, rather than the correct relations between the different variables which are measured in physics." *Ibid.,* p. 43. My interpretation denies this, i.e., I hold that Leibniz *is* interested primarily in setting up the new science of dynamics. A more recent paper, which has come to my attention since the writing of this paper, gives a thorough account of this example of Leibniz' from the physical standpoint (Carolyn litis, "Leibniz and the *Vis Viva* Controversy", *Isis,* 61, 2, 1971, pp. 21-35). Professor litis makes use of a distinction originated by Mach (*Science of Mechanics,* p. 365) in claiming that Leibniz was attempting to do two things in the controversy: (1) find a *measure* of *vis;* (2) claim that the universal sum of this *vis* is constant. She argues that Leibniz succeeded empirically in only the first of these tasks. However, she denies that Leibniz had good grounds for claiming that he had succeeded in the second of these tasks, since his grounds were merely metaphysical. On the other hand, it is my claim throughout this chapter that Leibniz *initiated* the *vis viva* controversy—which he perceived as the search for a conserved quantity —in the main on metaphysical grounds. Any other interpretation would not do justice to significant portions of Leibniz' total scientific enterprise. As to the claim that metaphysical grounds are not sufficient for Leibniz' establishing a search for a conservation principle, it is questionable that there are in fact *any other* sorts of sufficient grounds. Meyerson's extended research and argument has conclusively shown that the conservation principles present in physical science cannot be said to have been originated and confirmed as either *a priori* demonstrative, or *a posteriori* demonstrative. Their status can only be what might be called metaphysically "plausible": "every proposition stipulating identity in time appears to us as invested *a priori* with a high degree of probability. It finds our minds prepared, it seduces them, and is immediately adopted, unless contradicted by very evident facts. Perhaps it would be wise to apply to statements of this category, intermediary between the *a priori* and the *a posteriori,* a special term. We should propose, for lack of a better one, the term *plausible.* Therefore every proposition stipulating identity in time, every law of conservation, is plausible." (Meyerson, *op. tit.,* p. 148.) It seems clear to me that Leibniz' claims to have found a conserved quantity are justifiable, and explainable, on Meyerson's cogent analysis of the status of conservation principles in science. For for further comments, see the Appendix to this Chapter.

25. *Ibid.*

26. Leibniz notes that Galileo's law allows the computation of the velocities. The computation is as follows:

For a falling body which starts from rest, the equation is:

(1) $2gs=v^2$

We know the distance to be:

Distance Body A=^{S}A=4

Distance Body B=^{S}B=1.

Hence, given that:

(2) $2gs_A = V^2 = 2\ g\ (4) = V_A{}^2$; and

(3) $2gs_B = V_A{}^2 = 2\ g\ (1) = V_B{}^2$, then

(4) $V_A{}^2/\ V_B{}^2 = 2g(4)/\ 2\ g(1) = 4/1$

In terms of the *simple* velocity, the ratio is

$$\frac{V_A}{V_B} = \frac{\sqrt{V_A^2}}{\sqrt{V_B^2}} = \frac{\sqrt{4}}{\sqrt{1}} = \frac{2}{1},$$ as Leibniz notes.

I am indebted to Professor Lee Hart for his suggestion on this note.

27. This is a generalized version of something noted by Galileo: under certain circumstances the velocities and momenta of bodies may be measured by their "percussive force", i.e., the ability, say, of a freely falling cannonball to drive a stake into the ground (*Dialogues Concerning Two New Sciences,* trans., H. Crew and A. De Salvio (New York: Dover), pp. 269-70. For a discussion of Galileo's notion of force, see R. S. Westfall, "The Problem of Force in Galileo's Physics", in *Galileo Reappraised,* ed. C. L. Golino (Berkeley: U.C. Press, 1966), pp. 67-95. Leibniz' more general notion of measuring force by "violent effect" is compounded of this Galilean idea, and Huygen's notion of "elevating force", which is to be found in Huygens, *op. tit.,* Vol. IX, p. 463.

28. *Specimen Dynamicum,* GM VI, 118; Loemker, p. 297.

29. *Critical Thoughts,* G IV, 372; Loemker, p. 395.

30. What follows is a hypothetical reconstruction, using only the Galilean equations known to Leibniz, of how his mathematical reasoning might have gone.

31. Some of these equations are suggested by Loemker, p. 301.

32. G IV, 371-2; Loemker, p. 394.

33. *Specimen Dynamicum,* GM VI, 236f; *De Ipsa Natura,* G IV, 510f. The "passive" inertial resistance is, in some sense, active, insofar as it represents a *power* of the substance. Using this interpretation, Leibniz was able to claim that substance was always active, even in those cases where passive vis was greater than active *vis* in some particular substance.

34. *Specimen Dynamicum,* GM VI, 235-6; Loemker, p. 437.

35. *Cf.,* for example, to Bourget, August 5, 1715, G III, 580; Loemker, p. 663.

36. *Specimen Dynamicum,* GM VI, 235-6; Loemker, p. 437.

37. *Ibid.*

38. *Ibid.*

39. For a concise and clear description of the ensuing development, see D. W. Theobald's, *The Concept of Energy* (London: E. & F. N. Spon, 1966), pp. 35-51. Theobald's conclusion is that *vis viva,* the "quantity 1/2 mv^2, what we now call kinetic energy, was assumed to be conserved only in elastic collisions; and there was the prophetic claim of Leibniz that *vis viva* was not lost absolutely in an inelastic collision ... It was left to the eighteenth and nineteenth centuries to supply a unified mechanics and to substantiate Leibniz' hypothesis. During the process energy was to emerge as a powerful concept with far-reaching consequences ..." *Ibid.,* p. 37.

40. *Specimen Dynamicum,* GM VI, 236; Loemker, p. 438.

41. *Ibid.*

42. To De Voider, no date, G II, 156.